Mark Scheme (Results)

Summer 2016

Pearson Edexcel International GCSE
in Further Pure Mathematics Paper 1
(4PM0/01)

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2016
Publications Code 4PM0_01_1606_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme.

  Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks. Can only be awarded if the relevant method mark(s) has (have) been gained.
  - B marks: unconditional accuracy marks (independent of M marks)
- **Abbreviations**
  - cao – correct answer only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - eeoo – each error or omission
- **No working**

  If no working is shown then correct answers may score full marks

  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**

  If there is a wrong answer indicated always check the working and award any marks appropriate from the mark scheme.

  If it is clear from the working that the “correct” answer has been obtained from incorrect working, award 0 marks.

  Any case of suspected misread which does not significantly simplify the question loses two A (or B) marks on that question, but can gain all the M marks. Mark all work on follow through but enter A0 (or B0) for the first two A or B marks gained.

  If working is crossed out and still legible, then it should be given any appropriate marks, as long as it has not been replaced by alternative work.

  If there are multiple attempts shown, then all attempts should be marked and the highest score on a single attempt should be awarded.

- **Follow through marks**

  Follow through marks which involve a single stage calculation can be awarded without working since you can check the answer yourself, but if ambiguous do not award.

  Follow through marks which involve more than one stage of calculation can only be awarded on sight of the relevant working, even if it appears obvious that there is only one way you could get the answer given.

- **Ignoring subsequent work**

  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. incorrect cancelling of a fraction that would otherwise be correct.

  It is not appropriate to ignore subsequent work when the additional work essentially shows that the candidate did not understand the demand of the question.

- **Linear equations**

  Full marks can be gained if the solution alone is given, or otherwise unambiguously indicated in working (without contradiction elsewhere). Where the correct solution only is shown substituted, but not identified as the solution, the accuracy mark is lost but any method marks can be awarded.

- **Parts of questions**

  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded in another

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$$(x^2+bx+c)=(x+p)(x+q) \text{ where } |pq|=|c|$$

$$(ax^2+bx+c)=(mx+p)(nx+q) \text{ where } |pq|=|c| \text{ and } |mn|=|a|$$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to x=….

3. Completing the square:

$$\text{Solving } x^2+bx+c=\left(x\pm\frac{b}{2}\right)^2\pm q\pm c \text{ where } q\neq 0$$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1.

2. Integration:

Power of at least one term increased by 1.

Use of a formula:

Generally, the method mark is gained by

either quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication

from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show....

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

-

Question Number	Scheme	Marks
1(a)	Substitute $x = \pm 2$ or divide by $(x-2)$ Rem = 0	M1 A1 (2)
(b)	Use remainder theorem with $x = \pm 1, \pm 3$; remainder theorem again or inspection OR Divide $\mathrm{f}(x)$ by $x-2$, Factorise quadratic $(x-2)(x+3)(x-1)$ All 3 brackets must be shown.	M1M1 A1 (3) [5]

Notes

(a)
M1: for either substituting ±2 or attempting to divide by $(x-2)$
A1: for the remainder = 0
This is a show so please check that $\mathrm{f}(\pm 2) = (\pm 2)^3 - 7(\pm 2) + 6$ is seen for M1
and $8 - 14 + 6 = 0$ or $2^3 - 2 \times 7 + 6 = 0$ is seen for the A mark

ALT Using division
M1: minimally acceptable answer for the quotient for this mark is $x^2 + 2x \pm k$ If there is no evidence of inclusion of a term in x^2 somewhere in their division – M0
A1: correct quotient $(x-2)(x^2 + 2x - 3)$ **and** there must be a conclusion.
ie., therefore $(x - 2)$ is a factor oe.

(b)
In general, first M1 for finding one factor or dividing by $(x-2)$, second M1 for finding second factor.

M1: for remainder theorem OR by inspection OR divide by $(x-2)$ to give a quadratic factor OR by expanding and comparing coefficients.
Note: If there is no evidence of inclusion of a term in x^2 somewhere in their division – M0
Look for $x^2 + 2x \pm k$ to award M1

M1: for using remainder theorem again OR by inspection OR factorising the quadratic factor (refer to general guidance) OR by comparing coefficients
A1: for answer as shown

Note: $(x-2)(x-3)(x+1)$ with no working is M0M0A0

Question Number	Scheme	Marks
2(a)	$\left(1+3x^2\right)^{-\frac{1}{3}}=1+\left(-\frac{1}{3}\right)\left(3x^2\right)+\frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)}{2!}\left(3x^2\right)^2+\frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\left(-\frac{7}{3}\right)}{3!}\left(3x^2\right)^3...$	M1
	$=1-x^2+2x^4-\frac{14}{3}x^6...$	A1A1 (3)
(b)	$\mathrm{f}\left(x\right)=\left(1-kx^2\right)\left(1+3x^2\right)^{-\frac{1}{3}}$	
	$=\left(1-kx^2\right)\left(1-x^2+2x^4-\frac{14}{3}x^6\right)...$	M1
	$=1-kx^2-x^2+kx^4+2x^4+...$	M1
	$=1-\left(1+k\right)x^2+\left(k+2\right)x^4+...$	A1 (3)
(c)	$k=4$	B1 (1) [7]

Notes

(a)

M1: for using a binomial expansion at least up to the term in x^6. Each term, must have at least, the correct power of x and the correct denominator. Allow slips in $n(n-1)(n-2)$. The expansion must start with 1. Must see evidence of $3x^2$ used correctly at least once.

A1: for two correct algebraic terms simplified.

A1: for a fully correct simplified expansion all on one line.

(b)

M1: for setting their binomial expansion at least up to the term in x^4 from (a) multiplied by $\left(1-kx^2\right)$

M1: for multiplying out their expansion by $\left(1-kx^2\right)$ at least up to the term in x^4. There will be 5 terms in the expansion. Ignore any terms beyond x^4

A1: for a fully correct expansion (which need not be simplified)

(c)

B1: for $k=4$

Question Number	Scheme	Marks
3(a)	$AC^2 = 10^2 + 10^2$ or $\left(\frac{1}{2}AC\right)^2 = 5^2 + 5^2$	B1
	$AE^2 = 8^2 + 50\ (= 114)$	M1
	$AE = 10.67.. = 10.7$	A1 (3)
(b)	Required angle is between *EX* and the base where *X* is midpoint of *AB*	B1
	$\tan\theta = \frac{\text{ht}}{\frac{1}{2}AD} = \frac{8}{5}$	M1 (any trig function for angle)
	$\theta = 57.99... = 58°$	A1 (3) [6]

Notes

(a)

B1: for using Pythagoras theorem to find AC^2 or $\left(\frac{1}{2}AC\right)^2$

M1: for applying Pythagoras theorem **correctly** to find AE^2, using a side of 8 cm and their $\left(\frac{1}{2}AC\right)^2$

A1: for $AE = 10.7$

(Please refer to general guidance for rounding to significant figures)

(b)

B1: for identifying the required angle in **a correct triangle**. That is all that is required for this mark and can be gained by implication from subsequent correct work.

M1: for any acceptable trigonometry to find the required angle.

To use cos or sin they need the midpoint of *AB*, *DC*, *CB* or *AD* and the length from *E* to the midpoint to any of those sides is $\sqrt{89} = 9.43..$

A1: for $\theta = 58°$

(Please refer to general guidance for rounding to decimal places)

Beware: Candidates must identify the correct angle so finding 58° from using 9.43 as *AE* and 8 as the height will give the correct answer, but this is B0M0

Question Number	Scheme	Marks
4(a)	$S_2 = 2a + d = \frac{2}{3}(a + 4d)$	M1 (either)
	$S_4 = 2(2a + 3d) = a + 9d + 3$	A1 (both)
	$4a = 5d$	
	$a = d + 1$	
	(i) $d = 4$ (ii) $a = 5$	dM1A1A1 (5)
(b)	$S_{p+2} - S_p = t_{p+2} + t_{p+1}$	
	$5 + 4(p+1) + 5 + 4p = 110$	M1A1
	$14 + 8p = 110$	
	$p = 12$	A1 cso (3)
	Alt: Use difference of sums with formula for sum (M1 complete method, A1 correct equation A1 correct answer)	
		[8]

Notes

(a)

M1: for either a **correct** equation for S_2 **OR** S_4

A1: for **correct** equations for both S_2 **AND** S_4

dM1: for forming **and** attempting to solve **TWO** simultaneous equations in *a* and *d* only. This mark is dependent on the first method mark. Please check carefully that **both** equations are used to find *a* and *d*. $a = 5$ and $d = 4$ is a common answer coming from using only $4a = 5d$.

(i)

A1: for $d = 4$

(ii)

A1: for $a = 5$

(b)

M1: for the difference of S_{p+2} and S_p equated to the sum of t_{p+2} and t_{p+1}. Uses $a+(n-1)d$ for both and equates to 110, with an attempt to find *p*. The method must be complete for this mark.

A1: for fully correct substitution, so $5+4(p+2-1)+5+(p+1-1)=110$ is fine for this mark.

A1: for p=12 cso

Note: The final A mark is to be withheld from candidates who obtain a correct *a* and *d* from an incorrect method in part (a)

ALT

M1: for an attempt to find the difference of the summation formulae (using their *a* and *d*), equated to 110 with an attempt to find *p*. The summation formula must be correct for this mark.

$$S_{p+2} - S_p = \frac{p+2}{2}\left(2\times5+(p+2-1)4\right)-\frac{p}{2}\left(2\times5+(p-1)4\right)=110$$

$$(p+2)(7+2p)-p(2p+3)=110$$

$$8p+14=110$$

$$p=12$$

A1: for a fully correct substitution into $S_{p+2} - S_p$ with correct *a* and *d*.

A1: for $p = 12$ cso

Note: The final A mark is to be withheld from candidates who obtain a correct *a* and *d* from an incorrect method in part (a)

Question Number	Scheme	Marks
5(a)	$3(\sin x\cos\alpha+\cos x\sin\alpha)=5(\sin x\cos\alpha-\cos x\sin\alpha)$	M1
	$8\cos x\sin\alpha=2\sin x\cos\alpha$	A1
	$8\dfrac{\sin\alpha}{\cos\alpha}=2\dfrac{\sin x}{\cos x}$	dM1
	$\tan x=4\tan\alpha$	ddM1A1 (5)
(b)	$\tan 2y=4\tan 30$	M1A1
	$\tan 2y=2.30940.....$	
	$2y=66.586,\quad 246.58,\quad 426.58$	dM1 (any correct value)
	$y=\ 123°$	A1 (4) [9]

Notes

(a) In general, M marks;

1st M1 for using the given identity to expand $3\sin(x+\alpha)$ and $5\sin(x-\alpha)$.

Allow $5\sin(x-\alpha)=5\{\sin x\cos(-\alpha)+\cos x\sin(-\alpha)\}$

2nd M1 for dividing their expansion by either $\cos\alpha$ AND $\cos x$ or $\sin\alpha$ AND $\sin x$

This is dependent on the first M mark.

3rd M1 for using the identity for tan **This is dependent on BOTH previous M marks**

In general, A marks;

1st A1, for collecting like terms at the beginning or near the end.

2nd A1, for the correct answer and solution as given. You must see all three stages for the M marks as above so do not allow for example; $8\cos x\sin\alpha=2\sin x\cos\alpha\Rightarrow\tan x=4\tan\alpha$ This scores M1A1M0M0A0

M1: for using the given identity to expand $3\sin(x+\alpha)$ and $5\sin(x-\alpha)$.

A1: for simplifying the expansion to $8\cos x\sin\alpha=2\sin x\cos\alpha$

dM1: for rearranging their equation to $8\dfrac{\sin\alpha}{\cos\alpha}=2\dfrac{\sin x}{\cos x}$ oe.

ddM1: for using the given identity to convert their rearranged equation in terms of $\tan x$ and $\tan\alpha$

A1*: for achieving the given result.

There must be no errors in their work for the award of this mark.

(b)

M1: for using the **given** result from part (a) to substitute 2y for x, and 30° for α.

A1: for $\tan 2y = \frac{4\sqrt{3}}{3} = 2.30940...$ accept $\tan 2y = 2.3$

dM1: for any correct value for 2y (correct to 1 dp or better), or any correct valid value for y (This mark can implied from the correct answer) **Dependent on 1st M mark.**

A1: for $y = 123°$. Ignore extra values outside of the required range.

SC: $\tan 2y = 4\tan 30^{\circ} \Rightarrow y = 33^{\circ}$ implies M1A1M1A0

Note: You will see $\tan 2y = 4\tan 30^{\circ} \Rightarrow y = 123^{\circ}$ because candidates will leave the calculation in their calculators. This is full marks.

Question Number	Scheme	Marks
6(a)	$x^5 = 1024, \quad x = 4$	M1,A1 (2)
(b)	$7y - 3 = 3^4 = 81, \; y = 12$	M1,A1 (2)
(c)	$2\log_a 5 + 8\log_a 5 = 10$ or $\log_a 25 + 4\log_a 25 = 10$	M1
	$\log_a 5 = 1$ or $\log_a 25 = 2$	M1
	$a = 5$	A1 (3)
(d)	$\dfrac{1}{\log_7 b} - 2\log_7 b + 1 = 0$ (or change to base b)	M1
	$1 - 2(\log_7 b)^2 + \log_7 b = 0$	dM1
	$(2\log_7 b + 1)(\log_7 b - 1) = 0$	ddM1
	$\log_7 b = -\dfrac{1}{2} \quad b = 7^{-\frac{1}{2}} \quad (= 0.3779... = 0.378)$	A1
	$\log_7 b = 1 \quad b = 7$	A1 (5)
		[12]

Notes

(a)

M1: ‘undoes’ the log to write $x^5 = 1024$

A1: for $x = 4$

Award M1A1 for $x = 4$ seen only

(b)

M1: ‘undoes’ the log to achieve $7y - 3 = 3^4$ or $7y - 3 = 81 \Rightarrow y = ...$

A1: for $y = 12$

(c)

M1: the first M mark is for manipulating the logs so they can be combined into a single log.
Eg. $2\log_a 5 + 8\log_a 5 = 10$ or $\log_a 25 + 4\log_a 25 = 10$ or $\log_a 25 + \log_a 625^2 = 10$

M1: the second M mark is for combining the logs
Eg., $\log_a 5 = 1$ or $\log_a 25 = 2$ or $\log_a 9765625 = 10$

A1: $a = 5$ note $a = \pm 5$ is A0

SC: Because $5^2 = 25$ and $5^4 = 625$ you will see the following or similar;

$2+2\times 4=2+8=10\Rightarrow a=5$

Award full marks for a correct answer of *a* = 5 seen from this method.

(d)

M1: for changing the base of the log correctly either *b*

$$\log_b 7=\frac{\log_7 7}{\log_7 b} \text{ or } \log_7 b=\frac{\log_b b}{\log_b 7}$$

dM1: for forming a 3 term quadratic in either $\log_b 7$ or $\log_7 b$

Dependent on first M mark

$$1-2\left(\log_7 b\right)^2+\log_7 b=0 \quad \text{or} \quad \left(\log_b 7\right)^2+\log_b 7-2=0$$

ddM1: for solving their 3TQ **and** achieving two roots of their equation

Dependent on both M marks in (d)

$$\left(2\log_7 b+1\right)\left(\log_7 b-1\right)=0 \quad \text{or} \quad \left(\log_b 7-1\right)\left(\log_b 7+2\right)=0$$

A1: for **EITHER** $\log_7 b=-\frac{1}{2} \Rightarrow \quad b=7^{-\frac{1}{2}} \quad \left(=0.3779...=0.378\right)$

or $\log_b 7=-2\Rightarrow b^{-2}=7\Rightarrow b=7^{-\frac{1}{2}}=\frac{1}{\sqrt{7}}$ (accept awrt 0.378)

OR $\log_b 7 = 1$ so $b = 7$

A1: for **BOTH** correct answers

SC: some candidates are giving $1-2+1=0\Rightarrow \log_b 7-1=0\Rightarrow b=7$ Award first M mark only.

Beware of $2\log_7 b=0.5\log_b 7\Rightarrow b=\frac{1}{\sqrt{7}}$ This is M0.

A method using Trial and Improvement is M0

Question Number	Scheme	Marks
7(**a)**	Missing values -2.59, -1.17, 1.66 (B1B0 one correct; B1B1 all correct)	B1B1 (2)
(b)	All points plotted correctly within half of one square All points joined up in a smooth curve	B1ft B1ft (2)
(c)	$\log_2 7 = x$	
	$7 = 2^x$ $\quad 2^x - 4 = 3$	M1
	Draw line $y = 3$ or vertical from point on graph where $y = 3$ to x-axis	M1
	$\log_2 7 = 2.8$	A1 (3)
(d)	$2^x = 7 - 3x$	
	$y = 2^x - 4 = 3 - 3x$	M1A1
	Draw line $y = 3 - 3x$ $x = 1.4$	M1(their line) A1 (4) [11]
		-

Notes

(a)
B1: for one correct value
B1: for all values correct
(b)
B1ft: for all points plotted correctly within half of one square
B1ft: points joined up in a smooth curve
NOTE Part (c) and (d) must have evidence of their graph being used.
(c)
M1: for 'undoing' the log and substituting into $y = 2^x - 4 \Rightarrow y = 7 - 4 = 3$

OR $y = 2^x - 4 \Rightarrow 2^x = y + 4 \Rightarrow x = \log_2(y+4)$
$\log_2 7 = \log_2(y+4) \Rightarrow y = 3$

Note: an answer of 2.80.. without working or evidence of a mark or line on their graph is M0
M1: for drawing the line $y = 3$ or vertical from point on graph where $y = 3$ to x-axis or some evidence of using their graph from y = 3.
A1: for $x = 2.8$
(d)
M1; for attempting to re-arrange the equation to give $2^x - 4 = \pm k \pm 3x \quad k \neq 7$ or 0

A1: for $2^x - 4 = 3 - 3x$

M1: for drawing their '$y = 3 - 3x$' (look for intersections at (0, 3) and (1, 0) for the correct line) but it **must** be in the form $y = \pm k \pm 3x \quad k \neq 7$ or 0
A1: for $x = 1.4$

Note on Rounding

Some candidates are giving answers in (c) and (d) to 2 dp. Penalise only once (the first time) **PROVIDED** the answers given **both** round to 2.8 and 1.4 respectively. If answers given are for example, (c) 2.83 (d) 1.45, then this loses both A marks because part (c) is rounded incorrectly and part (d) rounds to 1.5 which is incorrect.

Question Number	Scheme	Marks
8(a)	(i) $\frac{2}{3}\mathbf{b}-\mathbf{a}$	B1
	(ii) $\overrightarrow{OE}=\overrightarrow{OA}+\frac{2}{5}\overrightarrow{AD}=\mathbf{a}+\frac{2}{5}\left(\frac{2}{3}\mathbf{b}-\mathbf{a}\right)=\frac{3}{5}\mathbf{a}+\frac{4}{15}\mathbf{b}$	M1A1
	(iii) $\overrightarrow{BE}=\overrightarrow{OE}-\overrightarrow{OB}=\frac{3}{5}\mathbf{a}+\frac{4}{15}\mathbf{b}-\mathbf{b}=\frac{3}{5}\mathbf{a}-\frac{11}{15}\mathbf{b}$	M1A1 (5)
(b)	$\overrightarrow{FE}=\overrightarrow{OE}-\overrightarrow{OF}=\frac{3}{5}\mathbf{a}+\frac{4}{15}\mathbf{b}-\lambda\mathbf{a}$	M1A1
	F, E, B collinear $\dfrac{\frac{3}{5}-\lambda}{\frac{4}{15}}=\dfrac{\frac{3}{5}}{-\frac{11}{15}}$	M1A1
	$\frac{3-5\lambda}{4}=\frac{3}{-11}$	
	$\lambda=\frac{9}{11}$	A1 (5)
	ALT $\overrightarrow{OF}+\overrightarrow{FB}=\overrightarrow{OB}$	M1
	$\lambda\mathbf{a}+\mu\left(-\frac{3}{5}\mathbf{a}+\frac{11}{15}\mathbf{b}\right)=\mathbf{b}$	A1
	$\mu=\frac{15}{11}$ $\quad\lambda=\frac{3}{5}\mu$	M1A1
	$\lambda=\frac{9}{11}$	A1 (5)
(c)	$\Delta OFB=5\text{ units}^2\Rightarrow\Delta OAB=\frac{11}{9}\times5\text{units}^2$	M1
	$\Delta OAD=\frac{2}{3}\Delta OAB=\frac{2}{3}\times\frac{55}{9}=\frac{110}{27}\text{ units}^2$	M1A1
	ALT $\frac{\text{area }\Delta OFB}{\text{area }\Delta OAD}=\frac{9/11}{2/3}=\frac{27}{22}$	M1
	$\text{area }\Delta OAD=\frac{22}{27}\times5=\frac{110}{27}$	M1A1 (3) [13]

Notes

(a) (i)

B1: for $\frac{2}{3}\mathbf{b}-\mathbf{a}$

(ii)

M1: for $\overrightarrow{OE}=\overrightarrow{OA}+\frac{2}{5}\overrightarrow{AD}$ (for the vector statement)

(or for any other valid path)

A1: $\overrightarrow{OE}=\frac{3}{5}\mathbf{a}+\frac{4}{15}\mathbf{b}$

(iii)

M1: for $\overrightarrow{BE}=\overrightarrow{OE}-\overrightarrow{OB}$ (for the vector statement)
(again for any other valid path)

A1: $\overrightarrow{BE}=\frac{3}{5}\mathbf{a}-\frac{11}{15}\mathbf{b}$

(b)

M1: for $\overrightarrow{FE}=\overrightarrow{OE}-\overrightarrow{OF}$

A1: for $\overrightarrow{FE}=\frac{3}{5}\mathbf{a}+\frac{4}{15}\mathbf{b}-\lambda\mathbf{a}$ $\left(=\mathbf{a}\left(\frac{3}{5}-\lambda\right)+\frac{4}{15}\mathbf{b}\right)$

M1: for using their $\overrightarrow{FE}$ and $\overrightarrow{BE}$ to form;

$$\frac{\frac{3}{5}-\lambda}{\frac{4}{15}}=\frac{\frac{3}{5}}{-\frac{11}{15}} \quad \text{or} \quad \frac{\frac{3}{5}-\lambda}{\frac{3}{5}}=\frac{\frac{4}{15}}{-\frac{11}{15}}$$

A1: for the correct equation in λ

A1: $\lambda=\frac{9}{11}$

ALT

M1: for $\overrightarrow{OF}+\overrightarrow{FB}=\overrightarrow{OB}$ oe

A1: for the correct expression in terms of λ and μ (or any other letter for the second constant)

M1: for comparing coefficients of λ and their μ

A1: for achieving μ and an expression for λ in terms of μ

A1: $\lambda=\frac{9}{11}$

(c)

M1: for stating and using that area of triangle $\Delta OAB = \frac{11}{9} \times$ area of $\Delta OFB \Rightarrow \Delta OAB = \frac{11}{9} \times 5$

Note: area of triangle OAB = the reciprocal of their $\lambda \times 5$

M1: for stating and using that area of $\Delta OAD = \frac{2}{3} \times$ area of ΔOAB

A1: area of triangle OAD = $\frac{110}{27}$

ALT 1

M1: for the ratio of areas of triangle OFB and triangle OAD as follows;

$$\frac{\text{area } \Delta OAB}{\text{area } \Delta OFB} = \frac{11}{9} \text{ and } \frac{\text{area } \Delta OAD}{\text{area } \Delta OAB} = \frac{2}{3} \Rightarrow$$

$$\frac{\text{area } \Delta OAD}{\text{area } \Delta OFB} = \frac{11}{9} \times \frac{2}{3} = \frac{22}{27}$$

M1: for $\frac{\Delta OAD}{5} = \frac{22}{27}$

A1: area of triangle OAD $\frac{110}{27}$

ALT 2

M1: for using $\frac{1}{2}ab\sin C$ on triangles OAD and OFB

Triangle OFB: $\frac{1}{2} \times \frac{9}{11} |\mathbf{a}| \times |\mathbf{b}| \times \sin\theta = 5$ **AND** Area $OAD = \frac{1}{2} \times |\mathbf{a}| \times \frac{2}{3} |\mathbf{b}| \times \sin\theta$

M1: for substituting . $\sin\theta = \frac{110}{9|\mathbf{a}||\mathbf{b}|}$ into $\Rightarrow$ Area $OAD = \frac{|\mathbf{a}||\mathbf{b}|}{3} \times \frac{110}{9|\mathbf{a}||\mathbf{b}|}$ $\left(= \frac{110}{27}\right)$

A1: area of triangle OAD $\frac{110}{27}$

Question Number	Scheme	Marks
9 (a) (i)	$\alpha+\beta=\frac{5}{3},\quad \alpha\beta=-\frac{4}{3}$	B1 Award in (i) or (ii)
	$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$	M1
	$\frac{\frac{25}{9}+\frac{8}{3}}{-\frac{4}{3}}=-\frac{49}{12}$	A1
	$\frac{\alpha}{\beta}\times\frac{\beta}{\alpha}=1$	B1
	$x^2-(\text{sum})x+\text{product}\quad(=0)$	M1
	$x^2-\left(-\frac{49}{12}\right)x+1\ (=0)$	
	$12x^2+49x+12=0$	A1 (6)
(ii)	$2\alpha+\beta+\alpha+2\beta=3\times\frac{5}{3}=5$	B1
	$(2\alpha+\beta)(\alpha+2\beta)=2\alpha^2+5\alpha\beta+2\beta^2$	M1,A1
	$=2(\alpha+\beta)^2+\alpha\beta,=2\times\frac{25}{9}-\frac{4}{3}=\frac{38}{9}$	
	$x^2-5x+\frac{38}{9}\quad(=0)$	M1
	$9x^2-45x+38=0$	A1 (5)
(b)	$\mathrm{f}(x)=3\left(x^2-\frac{5}{3}x\right)-4=3\left[\left(x-\frac{5}{6}\right)^2-\frac{25}{36}\right]-4$	M1
	$=3\left(x-\frac{5}{6}\right)^2-\frac{73}{12}$	A1A1 (3)
	(or by expanding $A(x+B)^2+C$ and equating coeffs)	
(c)	$\mathrm{f}(x)=-8\ \Rightarrow 3\left(x-\frac{5}{6}\right)^2-\frac{73}{12}=-8$	
	$3\left(x-\frac{5}{6}\right)^2=\frac{73}{12}-8<0\quad \therefore$ no values of x possible ie no real roots	M1A1cso (2)
	(or any other complete method M1; correct solution and conclusion A1)	[16]

Notes

(a) (i)

B1: for writing down the product and sum of the roots. This could be embedded in their calculations for sum and product.

M1: for forming the **correct** algebraic equation for the sum ie., $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$.

A1: for the correct sum = $-\frac{49}{12}$ oe

Note: $a^2 + b^2 = \frac{49}{9}$

B1: for product of roots = 1 (You may not see this explicitly, but can be implied if their constant in their formed equation

M1: for forming an equation using their sum and product

For this mark you **must see** $x^2 + (-\text{sum})x + (+\text{product})$ (=0)

A1: for the correct equation as shown including **= 0** Accept equivalent integer values, eg $24x^2 + 98x + 24 = 0$

(ii)

B1: for the sum of roots = 5

M1: for the algebraic product of roots. Multiplying out, simplifying to a minimally acceptable $m(\alpha+\beta)^2 + n\alpha\beta$ where $m \neq 0$ and $n \neq 0$

A1: for the product = $\frac{38}{9}$

M1: for forming an equation using their sum and product

A1: for the correct equation as shown **= 0**. **If =0 missing in part (i) do not penalise here again.** Accept equivalent integer values.

(b)

M1: for an attempt to complete the square. For this mark, they must take out 3 as the common factor in the term in x^2 and x (ignore the constant), and then complete the square (see General Guidance for minimally acceptable attempt)

A1: for two of A, B or C correct

A1: for A, B and C correct

ALT

M1: for $A(x+B)^2 + C = Ax^2 + 2ABx + B^2 + C \Rightarrow Ax^2 + 2ABx + B^2 + C \equiv 3x^2 - 5x - 4$

Must lead to values for A, B and C for this mark $\left(\Rightarrow A = 3, B = -\frac{5}{6}, C = -\frac{73}{12}\right)$

A1: for two of A, B or C correct

A1: for A, B and C correct

(c)

M1: for $3\left(x - \frac{5}{6}\right)^2 - \frac{73}{12} = -8 \Rightarrow 3\left(x - \frac{5}{6}\right)^2 = -\frac{23}{12}$ or using $b^2 - 4ac$ on the **given** f(x) + 8 = 0

A1: for a correct conclusion of eg., cannot find square root of negative number hence no real roots, or $b^2 - 4ac < 0$ hence no real roots. They **must** substitute correct values into $b^2 - 4ac$.

This A mark is **cso**

Question Number	Scheme	Marks
10 (a)	C is $(3,2)$ Or use ratio formula (correct) on either coord Both coords correct	M1 either correct A1 both (2)
(b)	Grad AB = $\frac{-2-4}{5-2}=-2$ Grad DC $=\frac{2-1}{3-1}=\frac{1}{2}$ $-2\times\frac{1}{2}=-1$ $\therefore$ perpendicular	B1 B1 B1 (3)
(c)	$y-1=\frac{1}{2}(x-1)$ $2y=x+1$	M1 A1 (2)
(d)	E is $(5,3)$	M1A1 (2)
(e)	$AB=\sqrt{3^2+6^2}=3\sqrt{5}$ $DE=\sqrt{4^2+2^2}=2\sqrt{5}$ or $CD=\sqrt{5}$ Area of kite $=\frac{1}{2}AB\times DE=\frac{1}{2}\times3\sqrt{5}\times2\sqrt{5}=15$ or $2\times\frac{1}{2}\times3\sqrt{5}\times\sqrt{5}=15$ Alt: Determinant method: Area $=\frac{1}{2}\begin{vmatrix}2 & 1 & 5 & 5 & 2\\4 & 1 & -2 & 3 & 4\end{vmatrix}$ $=\frac{1}{2}\left(2-2+15+20-(6-10+5+4)\right)=15$	M1 either A1 both M1A1 (4) M1A1 M1A1 (4) [13]

Notes

(a)
M1: for either correct x coordinate or y coordinate
A1: for both coordinates correct
Note: If you see either coord coming from an incorrect method M0
(b)
B1: for finding the gradient of AB
B1: for finding the gradient of DC
Do **not** accept vectors for gradients.
B1: for using the perpendicular rule to show that AB and DC are perpendicular, or stating that for gradients to be perpendicular, one must be the negative reciprocal of the other, with a conclusion.
eg., the negative reciprocal of -2 is $\frac{1}{2}$
Allow incorrect AB and CD here provided they are negatives reciprocals of each other.
(c)
M1: for using the formula with coordinates (1,1) or (3, 2) and a gradient of $\frac{1}{2}$ or their gradient of DC from (b) to write down the equation of the line. If they use $y = mx + c$ they must substitute x and y
correctly, their gradient of DC from (b) **and** find c
A1: for the correct equation in the correct form $2y = x + 1$.

(d)
M1: for either correct x coordinate or y coordinate
A1: for both coordinates correct
(e)
Method 1
M1: for finding either the length of AB (= $3\sqrt{5}$) or the length of DE or CD (using the given cords for D and their E. The Pythagoras must be correct if their E is incorrect.
A1: for both correct lengths of AB and DE or CD.
M1: for area of kite $\frac{1}{2} \times \text{'their'}\ AB \times \text{'their'}\ DE$
A1: for 15 (units^2)
Method 2
M1: for using the **CORRECT** formula for determinants with the given A, D, B, and 'their E'
A1: for a fully correct formula with correct coordinates
M1: for a correct calculation with the given A, D, B, and 'their E'
A1: for 15 (units^2)
Method 3 (General marking guidance for using a combination of areas)
M1: for attempting to calculate each individual area
A1: for correct individual areas (four triangles will be) 5, 5, 2.5, 2,5
Large rectangle (24) and 3 triangles (6,1.5,1.5)
M1: for a statement of the total area
A1: for 15 (units^2)

Pearson Education Limited. Registered company number 872828
with its registered office at 80 Strand, London WC2R 0RL